THERMAL INTERACTION OF PARALLEL GAS LINES WITH A NONADIABATIC REAL GAS FLOW

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Thermal interaction of parallel gas lines with a nonadiabatic real gas flow is investigated. The Poincaré method is applied to yield solutions useful for engineering design purposes.

When underground gas lines are located close together, their temperature fields interact, which undoubtedly affects the thermal and hydraulic conditions. A quantitative evaluation of this effect is of interest in connection with the design and operation of multiple gas lines. An analogous problem relating to the pumping of "hot" oils was examined in [1].

The present paper examines the thermal interaction of two parallel gas ducts with a nonadiabatic real gas flow on the basis of the general equations of gas dynamics (continuity, momentum, and energy) [1, 2]:

$$\gamma w_n f_n = G_n, \quad d\left(\frac{p_n}{\gamma} + \frac{w_n^2}{2g} + Z_n\right) = -dH_n,$$
$$d\left(\frac{w_n^2}{2g} + Z_n + \frac{i_n}{A}\right) = dU_n/AG_n, \quad n = 1,2. \tag{1a}$$

The equation of state is assumed to be [3]:

$$(P_n v_n)/(RT_n) = 1 + \frac{9}{128}\frac{P_n}{P_c}\frac{T_c}{T_n}\left(1 - 6\frac{T_c^2}{T_n^2}\right) = Z_0(P_n, T_n), \tag{1b}$$

which gives good results in the supercritical temperature region at moderate pressures $(0\text{-}100)\cdot 10^5\ \mathrm{N/m^2}$. Neglecting variations of velocity head and geometric height [2] in (1a) and taking into account the known thermodynamic relation for enthalpy, we obtain:

$$\frac{d\pi_1}{d\varkappa} = -\mu_1\frac{\tau_1}{\pi_1}, \quad \frac{d\pi_2}{d\varkappa} = -\mu_2\frac{\tau_2}{\pi_2},$$
$$\frac{d\tau_1}{d\varkappa} = \varepsilon\mu_1\left(1 - \frac{18}{\tau_1^2}\right)\frac{\tau_1}{\pi_1} - a_1(\tau_1 - \tau_0) + b_1(\tau_2 - \tau_0),$$
$$\frac{d\tau_2}{d\varkappa} = \varepsilon\mu_2\left(1 - \frac{18}{\tau_2^2}\right)\frac{\tau_2}{\pi_2} - a_2(\tau_2 - \tau_0) + b_2(\tau_1 - \tau_0), \tag{2}$$

$$\pi_1 = \pi_{1i}, \ \pi_2 = \pi_{2i}, \ \tau_1 = \tau_{1i}, \ \tau_2 = \tau_{2i} \text{ when } \varkappa = 0,$$

$$\pi_n = \frac{P_n}{P_c}, \quad \tau_n = \frac{T_n}{T_c}, \quad \varkappa = \frac{x}{L}, \quad \varepsilon = \frac{9}{128}\frac{AR}{c_p},$$

$$\mu_n = \frac{\lambda_n Z_{0n} RT_c L}{2gD_n P_c^2}\left(\frac{G_n}{f_n}\right)^2,$$

$$a_1 = \left[\frac{2\Pi\lambda_{\text{rp}}L}{G_1 c_p}\ln\frac{4h_2}{D_2}\right]\times$$
$$\times\left[\ln\frac{4h_1}{D_1}\ln\frac{4h_2}{D_2} - \ln^2\sqrt{\frac{l^2 + (h_1 + h_2)^2}{l^2 + (h_1 - h_2)^2}}\right]^{-1}, \ a_2 = a_1\frac{G_1}{G_2},$$

$$b_1 = \left[\frac{2\Pi\lambda_{\text{rp}}L}{G_1 c_p}\ln\sqrt{\frac{l^2 + (h_1 + h_2)^2}{l^2 + (h_1 - h_2)^2}}\right]\times$$
$$\times\left[\ln\frac{4h_1}{D_1}\ln\frac{4h_2}{D_2} - \ln^2\sqrt{\frac{l^2 + (h_1 + h_2)^2}{l^2 + (h_1 - h_2)^2}}\right]^{-1}, \ b_2 = b_1\frac{G_1}{G_2}.$$

The subscript 1 relates to the hotter gas line. We shall seek a solution of (2) by the Poincaré method [4]. For real gases the parameter ε is very small (of the order of 10^{-2}). The distributions of temperature φ_n and pressure ψ_n along the length of the line for a perfect gas with $\varepsilon = 0$ are obtained from solving (2):

$$\varphi_1(x) = \tau_0 + (\tau_{1i} - \tau_0)\exp(K_1 x) - \frac{K_2 + a_2}{b_2(K_1 - K_2)}[(\tau_{2i} - \tau_0)(K_1 + a_2) - (\tau_{1i} - \tau_0) b_2] \times [\exp(K_1 x) - \exp(K_2 x)],$$

$$\varphi_2(x) = \tau_0 + (\tau_{1i} - \tau_0)\frac{b_2}{K_1 + a_2}\exp(K_1 x) - \frac{[(\tau_{2i} - \tau_0)(K_1 + a_2) - (\tau_{1i} - \tau_0) b_2]}{(K_1 - K_2)(K_1 + a_2)} \times [(K_2 + a_2)\exp(K_1 x) - (K_1 + a_2)\exp(K_2 x)],$$

$$\psi_n(x) = \sqrt{\pi_{ni}^2 - 2\mu_n \int_0^x \varphi_n(x)\,dx},$$

$$K_{1,2} = -\frac{a_1 + a_2}{2} \pm \sqrt{\left(\frac{a_1 - a_2}{2}\right)^2 + b_1 b_2}. \qquad (3)$$

The Poincaré solution of system (2) [4] is sought in the form of linear combinations

$$\tau_1 = \varphi_1 + \varepsilon\bar{\tau}_1^{(1)}, \quad \tau_2 = \varphi_2 + \varepsilon\bar{\tau}_2^{(1)}, \quad \pi_1 = \psi_1 + \varepsilon\bar{\pi}_1^{(1)}, \quad \pi_2 = \psi_2 + \varepsilon\bar{\pi}_2^{(1)}. \qquad (4)$$

Omitting the intermediate steps, we give the equation for correcting temperature and pressure to take account of the differences between a real and a perfect gas:

$$\bar{\tau}_1^{(1)} = \frac{(a_2 + K_1)\exp(K_1 x)}{b_2(K_1 - K_2)}\left[b_2\mu_1\int_0^x \frac{\varphi_1}{\psi_1}\left(1 - \frac{18}{\varphi_1^2}\right)\frac{dx}{\exp(K_1 x)} - (a_2 + K_2)\mu_2\int_0^x \frac{\varphi_2}{\psi_2}\left(1 - \frac{18}{\varphi_2^2}\right)\frac{dx}{\exp(K_2 x)}\right] + \frac{(a_2 + K_2)\exp(K_2 x)}{b_2(K_1 - K_2)}\left[(a_2 + K_1)\mu_2\int_0^x \frac{\varphi_2}{\psi_2}\left(1 - \frac{18}{\varphi_2^2}\right)\frac{dx}{\exp(K_2 x)} - b_2\mu_1\int_0^x \frac{\varphi_1}{\psi_1}\left(1 - \frac{18}{\varphi_1^2}\right)\frac{dx}{\exp(K_2 x)}\right],$$

$$\bar{\tau}_2^{(1)} = \frac{\exp(K_1 x)}{K_1 - K_2}\left[b_2\mu_1\int_0^x \frac{\varphi_1}{\psi_1}\left(1 - \frac{18}{\varphi_1^2}\right)\frac{dx}{\exp(K_1 x)} - (a_2 + K_2)\mu_2\int_0^x \frac{\varphi_2}{\psi_2}\left(1 - \frac{18}{\varphi_2^2}\right)\frac{dx}{\exp(K_2 x)}\right] + \frac{\exp(K_2 x)}{K_1 - K_2}\left[(a_2 + K_1)\mu_2\int_0^x \frac{\varphi_2}{\psi_2}\left(1 - \frac{18}{\varphi_2^2}\right)\frac{dx}{\exp(K_2 x)} - b_2\mu_1\int_0^x \frac{\varphi_1}{\psi_1}\left(1 - \frac{18}{\varphi_1^2}\right)\frac{dx}{\exp(K_2 x)}\right],$$

$$\bar{\pi}_n^{(1)} = \mu_n \exp\left(\mu_n \int_0^{\varkappa} \frac{\varphi_n}{\psi_n^2}\, d\varkappa\right)\left[-\int_0^{\varkappa} \frac{\bar{\tau}_n^{(1)}}{\psi_n} \exp\left(-\mu_n \int_0^{\varkappa} \frac{\varphi_n}{\psi_n^2}\, d\varkappa\right) d\varkappa\right]. \quad (5)$$

The integrations in (5) can be performed numerically with the requisite degree of accuracy. Let us examine the gas temperature distribution along the interacting gas lines in terms of the solutions obtained.

The following cases are of practical interest:

1. $\tau_{1i} = \tau_{2i} = \tau_0$; $\pi_{1i} = \pi_{2i} = \pi_i$. Expressing the integrals in Eqs. (5) in terms of the Cramp function [5]

$$\int_0^U \exp(-\xi^2\varkappa^2)\, d\varkappa = \frac{\sqrt{\Pi}}{2\xi}\, \mathrm{erf}(\xi U) \quad (6)$$

and neglecting corrections for the pressure variation due to the gas being thermodynamically imperfect and due to thermal interaction (because of their smallness), we may represent the temperature distribution along the interacting gas lines in the form:

$$\tau_1(\varkappa) = \varphi_1(\varkappa) - \frac{I_0\sqrt{\Pi}}{2b_2(K_1-K_2)}\left\{(a_2+K_1)\left[\frac{b_2}{\sqrt{q_{11}}}\exp(q_{11}\pi_i^2)\times\right.\right.$$
$$\times\left[\mathrm{erf}(\pi_i\sqrt{q_{11}}) - \mathrm{erf}(\pi_1\sqrt{q_{11}})\right] - \frac{(a_2+K_2)}{\sqrt{q_{12}}}\exp(q_{12}\pi_2^2)\times$$
$$\left.\times\left[\mathrm{erf}(\pi_i\sqrt{q_{12}}) - \mathrm{erf}(\pi_2\sqrt{q_{12}})\right]\right] + (a_2+K_2)\left[\frac{a_2+K_1}{\sqrt{q_{22}}}\exp(q_{22}\pi_2^2)\times\right.$$
$$\times\left[\mathrm{erf}(\pi_i\sqrt{q_{22}}) - \mathrm{erf}(\pi_2\sqrt{q_{22}})\right] -$$
$$\left.\left.-\frac{b_2}{\sqrt{q_{21}}}\exp(q_{21}\pi_i^2)\left[\mathrm{erf}(\pi_i\sqrt{q_{21}}) - \mathrm{erf}(\pi_1\sqrt{q_{21}})\right]\right]\right\},$$

$$\tau_2(\varkappa) = \varphi_2(\varkappa) - \frac{I_0\sqrt{\Pi}}{2(K_1-K_2)}\left\{\left[\frac{b_2}{\sqrt{q_{11}}}\exp(q_{11}\pi_i^2)\left[\mathrm{erf}(\pi_i\sqrt{q_{11}}) -\right.\right.\right.$$
$$\left.\left.-\mathrm{erf}(\pi_1\sqrt{q_{11}})\right] - \frac{(a_2+K_2)}{\sqrt{q_{12}}}\exp(q_{12}\pi_2^2)\left[\mathrm{erf}(\pi_i\sqrt{q_n}) - \mathrm{erf}(\pi_2\sqrt{q_{12}})\right]\right] +$$
$$+\left[\frac{(a_2+K_1)}{\sqrt{q_{22}}}\exp(q_{22}\pi_2^2)\left[\mathrm{erf}(\pi_i\sqrt{q_{22}}) - \mathrm{erf}(\pi_2\sqrt{q_{22}})\right] - \frac{b_2}{\sqrt{q_{21}}}\exp(q_{21}\pi_i^2)\times\right.$$
$$\left.\left.\times\left[\mathrm{erf}(\pi_i\sqrt{q_{21}}) - \mathrm{erf}(\pi_1\sqrt{q_{21}})\right]\right]\right\}, \quad (7)$$

$$\pi_n = \sqrt{\pi_i^2 - 2\mu_n\tau_0\varkappa},$$

$$I_0 = \frac{9}{128}\,\frac{AR}{c_p}\left(\frac{18}{\tau_0^2} - 1\right),$$

$$q_{11} = \frac{|K_1|}{2\mu_1\tau_0}, \quad q_{12} = \frac{|K_1|}{2\mu_2\tau_0}, \quad q_{21} = \frac{|K_2|}{2\mu_1\tau_0}, \quad q_{22} = \frac{|K_2|}{2\mu_2\tau_0}.$$

2. The inequalities $|\tau_{1i} - \tau_0| \ll 1$, $|\tau_{2i} - \tau_0| \ll 1$, $\tau_0 > 1$ usually hold, and we may therefore use (7) even for $\tau_{ni} \neq \tau_0$.

The heat losses of parallel interacting gas lines depend, amongst other factors, on the distance l between them and the thermal conductivity of the ground λ_{gr}. The heat losses of isolated and interacting gas lines are related as follows [1]:

$$\beta_1 = \left(1 - \theta \ln\sqrt{1 + \left(\frac{2h}{l}\right)^2} \Big/ \ln\frac{4h}{D_2}\right)\times \quad (8)$$

$$\times\left(1-\ln^2\sqrt{1+\left(\frac{2h}{l}\right)^2}\Big/\ln\frac{4h}{D_1}\ln\frac{4h}{D_2}\right)^{-1},$$

$$\beta_2=\left(1-\theta^{-1}\ln\sqrt{1+\left(\frac{2h}{l}\right)^2}\Big/\ln\frac{4h}{D_1}\right)\times$$

$$\times\left(1-\ln^2\sqrt{1+\left(\frac{2h}{l}\right)^2}\Big/\ln\frac{4h}{D_1}\ln\frac{4h}{D_2}\right)^{-1}, \qquad (8)\ \text{(cont'd)}$$

$$\theta=(\tau_2-\tau_0)/(\tau_1-\tau_0).$$

The effect of thermal interaction of gas lines of diameter 1000 and 700 mm for h = 1.2 m is shown in Fig. 1. It follows from an analysis of (8) and from Fig. 1, that as l decreases (λ_{gr} = const) the losses of the hotter line β_1 decrease and reach a minimum when

$$l_x=2h\left[\left(\frac{4h}{D_2}\right)^{2\left[\frac{1}{\theta}-\sqrt{\frac{1}{\theta^2}-\frac{\ln(4h/D_1)}{\ln(4h/D_2)}}\right]}-1\right]^{-1/2}, \qquad (9)$$

determined from the condition $d\beta_1/dl = 0$. Then $\beta_2 = 0.5$, i.e., the heat losses of the colder line due to interaction are half those of an isolated duct at temperature τ_2.

When $l < l_x$, β_1 increases, and reaches a maximum when $l_{min} = (D_1 + D_2)/2$; β_2 then decreases and at a distance

$$l_0=2h\left[\left(\frac{4h}{D_1}\right)^{2\theta}-1\right]^{-1/2} \qquad (10)$$

becomes equal to zero, i.e., locating the colder line at distance l_0 does not affect the heat losses of the hotter line, which are then equal to those of an isolated duct at temperature τ_1. This may occur if the colder line coincides with an isotherm of the hotter line at temperature τ_2 [1]. When $l < l_0$, β_2 becomes negative, i.e., the colder line is heated due to losses from the hotter one. When $l <$ 20 m (λ_{gr} up to 2.8 kJ/m · sec · degree), there is no thermal interaction, and the heat losses of the system $\beta_0 = \beta_1 + \beta_2$ tend to a maximum.

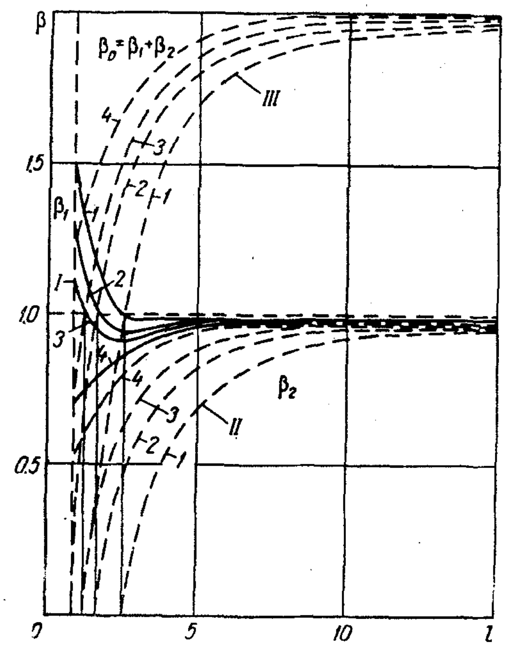

Fig. 1. Thermal interaction of two gas lines (h = 1.2 m): I and II) dimensionless heat losses of hotter line (D_1 = 1000 mm) and of colder line (D_2 = 700 mm); III) total heat losses ($\beta_0 = \beta_1 + \beta_2$); 1) θ = 0.2; 2) 0.4; 3) 0.6; 4) 1.0;

To obtain a quantitative evaluation of the effect of thermal interaction on the temperature distribution along parallel gas lines with a nonadiabatic real gas flow (methane), calculations were performed for the following conditions: $P_i = 53.9 \cdot 10^5$ N/m²; $G_1 = 9.0 \cdot 10^5$ kg/hr; $G_2 = 3.25 \cdot 10^5$ kg/hr; D_1 = 1.0 m; D_2 = 0.7 m; L = 120 km; $h_1 = h_2$ = 1.2 m; λ_1 = 0.010; λ_2 = 0.013; $P_c = 44.9 \cdot 10^5$ N/m²; T_c = 190.5°K; C_p = 2.219 kj/kg · degree; R = 53 m/degree; Z_{0h} = 0.93.

Variation of the real gas temperature along the interacting gas lines, was calculated from (7) for various values of λ_{gr}, distances between duct axes l, and initial gas temperatures τ_{1i}, τ_{2i} (Figs. 2, 3). For comparison, the temperature distribution of a thermodynamically perfect gas in interacting gas lines was calculated from (3) and from the Shukov formulas and from [6] for a perfect and the corresponding real gas without taking account of thermal interaction.

An analysis of the cases most important in practice shows that:

1. When $l < l_0$ ($\tau_{1i} = \tau_{2i}$) the temperature curve (curve 4) of the hotter gas line for a real gas with allowance for thermal interaction, passes below the curves for a perfect and a real gas 1 and 3. This is explained by addition of the interaction effects ($\beta_1 > 1$) and by departure from perfect thermodynamic conditions (Fig. 2). The temperature curve of the colder line for a real gas (8) consists of two parts – one (to the point of inflection) corresponding to heating and the other (after that) to cooling; curve 8 passes below 7 for a perfect gas with interaction, and above curve 5. The latter observation is explained by the opposing action of the Joule-Thomson and interaction effects: heating of the gas due to thermal interaction ($\beta_2 < 0$) and cooling due to thermodynamic imperfection. It may be seen from Fig. 2 that in this region ($l < l_0$) increase in temperature due to interaction predominates over the Joule-Thomson effect, and the temperature of the real gas remains considerably higher than that of a perfect gas in the absence of interaction.

2. When $l_x > l > l_0$ ($\tau_{1i} = \tau_{2i}$) the temperature curve 4 passes below curve 2 due to predominance of the Joule-Thomson effect over heating of the gas due to interaction (Fig. 3a).

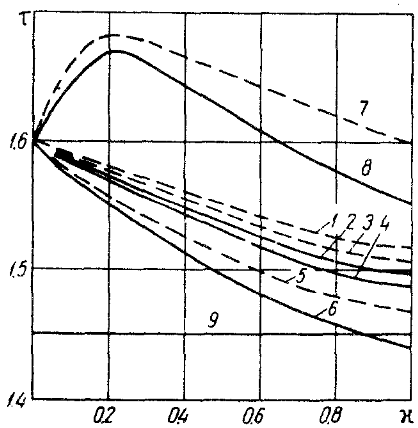

Fig. 2. Dependence of dimensionless temperature of the gas, τ, on $\varkappa$ for $\lambda_{gr} = 0.913$, $l = 0.9$ m: 1, 2, 3, 4 and 5, 6, 7, 8) according to Shukov's formulas [6], (3), and (7) for $D_1 = 1000$ mm and $D_2 = 700$ mm, respectively; 9) dimensionless ground temperature.

3. When $l_x > l > l_0$ ($\tau_{1i} > \tau_{2i}$) the temperature curve of the colder line for a real gas (8) is higher than that for a perfect gas in the absence of interaction (5), which is explained by the predominance of heating due to interaction over cooling due to the Joule-Thomson effect (Fig. 3b); with increase of λ_{gr}, the effect of interaction on the thermal conditions decreases.

Thus, for maximum utilization of the carrying capacity of the hotter gas line and improved thermal conditions in the colder one (avoidance of hydrate formation, etc.), the distance between them should be within the range $l_x > l > l_0$ for the corresponding value of Θ. When this spacing is impossible for safety reasons, l should be chosen in the range 10-12 m (Fig. 1). An analysis of the derived formulas and the calculated data (Fig. 2) indicates that thermal interaction of parallel gas lines produces a considerable change in the thermal conditions in each of them. This should be taken into account in cases when it is important to know the temperature of a real piped gas (condensation of hydrocarbons and water, hydrate formation, choice of insulation, etc.).

Thermal interaction can be put to good use, for instance, to provide partial cooling of the gas in the pipe itself.

NOTATION

T_1, T_2 – outer surface temperatures of gas lines; T_0 – temperature of surrounding ground (under normal thermal conditions); h_1, h_2 – depths of lines below ground (to axes); l – distance between axes; D_1, D_2 – outsider diameters of gas lines; λ_{gr} – thermal conductivity of ground; G – mass flow rate; f – cross-sectional area of gas lines; L – length of line; P – absolute pressure; λ – friction coefficient; v – specific volume; Z_0 – compressibility factor; R – gas constant; P_c, T_c – respectively, critical pressure and temperature; c_p – specific heat at constant pressure; i – enthalpy; H – loss of heat due to friction; w – gas velocity; U – internal energy; A – thermal equivalent of mechanical work; g – acceleration due to gravity.

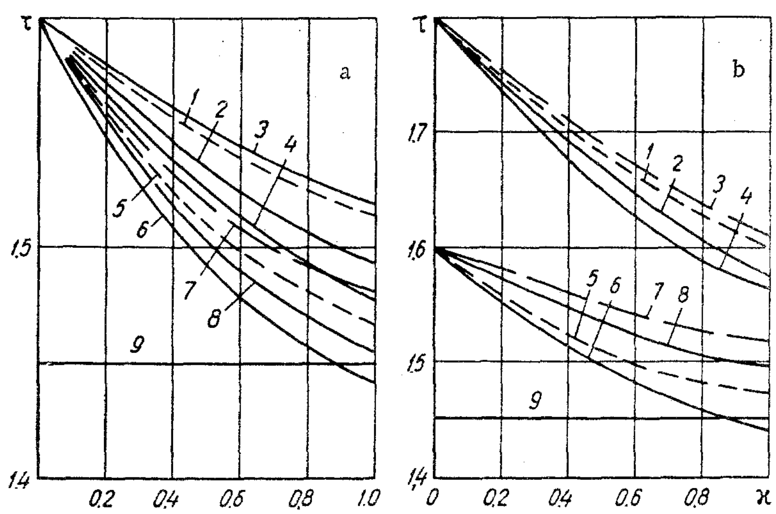

Fig. 3. Dependence of gas temperature τ on $\varkappa$ for $\tau_{1i} = \tau_{2i}$ (a), $\tau_{1i} > \tau_{2i}$ (b), $\lambda_{gr} = 0.913$, $l = 2$ m: 1-9 see Fig. 2.

REFERENCES

1. V. I. Chernikin, Pumping Viscous and Congealing Oils [in Russian], Gostoptekhizdat, 1958.
2. I. A. Charnyi, Basic Gas Dynamics [in Russian], Gostoptekhizdat, 1961.
3. M. P. Vukalovich and I. I. Novikov, Equation of State of a Real Gas [in Russian], Gostoptekhizdat, 1948.
4. I. S. Berezin and N. P. Zhidkov, Numerical Methods [in Russian], 1, 2, Fizmatgiz, 1962.
5. B. I. Segal and K. A. Semendyaev, Five-Figure Mathematical Tables [in Russian], Izd. AN SSSR, 1950.
6. B. V. Shalimov, IFZh, no. 2, 1963.

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